MATLAB SOFTWARE FOR SPATIAL PANELS

J. Paul Elhorst*

Faculty of Economics and Business, University of Groningen, P.O. Box 800,

9700 AV Groningen, The Netherlands, Phone: +31 50 363 3893, Fax: +31 50 363 7337,

j.p.elhorst@rug.nl

Abstract

Elhorst (2003, 2010a) provides Matlab routines to estimate spatial panel data models at his Web site. This paper extends these routines to include the bias correction procedure proposed by Lee and Yu (2010a) if the spatial panel data model contain spatial and/or time-period fixed effects, the direct and indirect effects estimates of the explanatory variables proposed by LeSage and Pace (2009), and a selection framework to determine which spatial panel data model best describes the data. To demonstrate these routines in an empirical setting, a demand model for cigarettes is estimated based on panel data from 46 U.S. states over the period 1963 to 1992.

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1. Introduction

In recent years, the spatial econometrics literature has exhibited a growing interest in the specification and estimation of econometric relationships based on spatial panels. This interest can be explained by the fact that panel data offer researchers extended modeling possibilities as compared to the single equation cross-sectional setting, which was the primary focus of the spatial econometrics literature for a long time.

To estimate spatial panel data models, Elhorst (2003, 2010a) provides Matlab routines at his website www.regroningen.nl/elhorst for the fixed effects and random effects spatial lag model, as well as the fixed effects and random effects spatial error model. The objective of this paper is to extend these routines for two recent developments in the spatial econometrics literature. First, Lee and Yu (2010a) show that the direct approach of estimating a model with spatial fixed effects, as set out in Elhorst (2003, 2010a), will yield an inconsistent parameter estimate of the variance parameter (σ^2) if N is large and T is small, and inconsistent estimates of all parameters of a model with spatial and time-period fixed effects if both N and T are large. To correct for this, they propose a bias correction procedure based on the parameter estimates of the direct approach. The second development is the increasing attention for direct and indirect effects estimates of the independent variables in both the spatial lag model and the spatial Durbin model (LeSage and Pace, 2009). Direct effects estimates measure the impact of changing an independent variable on the dependent variable of a spatial unit. This measure includes feedback effects, i.e., impacts passing through neighboring units and back to the unit that instigated the change. Indirect effects estimates measure the impact of changing an independent variable in a particular unit on the dependent variable of all other units.

A second objective of this paper is to demonstrate these extended routines in an empirical setting. Today a (spatial) econometric researcher has the choice of many models. First, he should ask himself whether or not, and, if so, which type of spatial interaction effects should be accounted for: (1) a spatially lagged dependent variable, (2) spatially lagged independent variables, (3) a spatially autocorrelated error term, or (4) a combination of these. Second, he should ask himself whether or not spatial-specific and/or time-specific effects should be accounted for and, if so, whether they should be treated as fixed or as random effects. Two routines have been developed and made available consisting of

different statistical tests to help the researcher choose among different alternatives. The first routine provides (robust) LM tests, generalizing the classic LM-tests proposed by Burridge (1980) and Anselin (1988) and the robust LM-tests proposed by Anselin et al. (1996) from a cross-sectional setting to a spatial panel setting. This generalization is based on Debarsy and Ertur (2010), who consider the testing of a spatially lagged dependent variable and/or spatial error autocorrelation in a model with spatial fixed effects.[1] The second routine contains a framework to test the spatial lag, the spatial error model, and the spatial Durbin model against each other, as well as a framework to choose among fixed effects, random effects or a model without fixed/random effects. To illustrate a model selection procedure based on these routines, we estimate a demand model for cigarettes based on panel data from 46 U.S. states over the period 1963 to 1992. This data set is taken from Baltagi (2005) and has been used for illustration purposes in other studies as well.

The setup of this paper is as follows. In Section 2, we set out three panel data models to put spatial dependence into practice. Next, we present the bias correction procedures and the direct and indirect effects estimates of these models in mathematical form. In Section 3, we report and discuss the results of our empirical analysis, and in Section 4 we offer our conclusions.

2. Model specification

As pointed out by Anselin et al. (2008), when specifying spatial dependence among the observations, a spatial panel data model may contain a spatially lagged dependent variable, or the model may incorporate a spatially autoregressive process in the error term. The first model is known as the spatial lag model and the second as the spatial error model. A third model, advocated by LeSage and Pace (2009), is the spatial Durbin model that contains a spatially lagged dependent variable and spatially lagged independent variables.

Formally, the spatial lag model is formulated as

$$y_{it} = \delta \sum_{j=1}^{N} w_{ij} y_{jt} + \alpha + x_{it}\beta + \mu_i \,(\text{optional}) + \lambda_t \,(\text{optional}) + \varepsilon_{it}, \tag{1}$$

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[1] Baltagi et al. (2003) are the first to consider the testing of spatial interaction effects in a spatial panel data model. They derive a joint LM test which simultaneously tests for spatial error autocorrelation and spatial random effects, as well as two conditional tests which test for one of these extensions assuming the presence of the other.

where y_{it} is the dependent variable for cross-sectional unit i at time t (i=1, ..., N; t=1, ..., T). The variable $\sum_j w_{ij} y_{jt}$ denotes the interaction effect of the dependent variable y_{it} with the dependent variables y_{jt} in neighboring units, where w_{ij} is the i,j-th element of a pre-specified nonnegative N×N spatial weights matrix W describing the arrangement of the spatial units in the sample. The response parameter of these endogenous interaction effects, δ, is assumed to be restricted to the interval $(1/r_{min}, 1)$, where r_{min} equals the most negative purely real characteristic root of W after this matrix has been row-normalized (see LeSage and Pace, 2009, pp. 88-89 for mathematical details).[2] α is the constant term parameter. x_{it} a 1×K vector of exogenous variables, and β a matching K×1 vector of fixed but unknown parameters. ε_{it} is an independently and identically distributed error term for i and t with zero mean and variance σ^2, while μ_i denotes a spatial specific effect and λ_t a time-period specific effect. Spatial specific effects control for all space-specific time-invariant variables whose omission could bias the estimates in a typical cross-sectional study, while time-period specific effects control for all time-specific effects whose omission could bias the estimates in a typical time-series study (Baltagi, 2005). If μ_i and/or λ_t are treated as fixed effects, the intercept α can only be estimated under the condition(s) that $\sum_i \mu_i = 0$ and $\sum_t \lambda_t = 0$. An alternative and equivalent formulation is to drop the intercept from the model and to abandon one of these two restrictions (see Hsaio, 2003, p. 33).

In the spatial error model, the error term of unit i, φ_{it}, is taken to depend on the error terms of neighboring units j according to the spatial weights matrix W and an idiosyncratic component ε_{it}, or formally

$$y_{it} = \alpha + x_{it}\beta + \mu_i \text{(optional)} + \lambda_t \text{(optional)} + \phi_{it}, \quad \phi_{it} = \rho \sum_{j=1}^{N} w_{ij}\phi_{it} + \varepsilon_{it}, \tag{2}$$

where ρ is called the spatial autocorrelation coefficient.

To test whether the spatial lag model or the spatial error model is more appropriate

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[2] Kelejian and Prucha (2010) point out that the normalization of the elements of the spatial weights matrix by a different factor for each row as opposed to a single factor is likely to lead to a misspecification problem. For this reason, they propose a normalization procedure where each element of W is divided by its largest characteristic root. This normalization procedure is left aside in this paper because of both assumption 1' and footnote 21 in Lee and Yu (2010a).

to describe the data than a model without any spatial interaction effects, one may use Lagrange Multiplier (LM) tests for a spatially lagged dependent variable and for spatial error autocorrelation, as well as the robust LM-tests which test for the existence of one type of spatial dependence conditional on the other. A mathematical derivation of these tests for a spatial panel data model with spatial fixed effects can be found in Debarsy and Ertur (2010). These tests are based on the residuals of the non-spatial model with spatial fixed effects and follow a chi-squared distribution with one degree of freedom. If a non-spatial model is estimated without any fixed effects or a non-spatial model with both spatial and time-period fixed effects, the residuals of these models can be used instead (Elhorst, 2010a). Since the outcomes of these tests depend on which effects are included, it is recommended to carry out these LM tests for different panel data specifications. A Matlab routine (LMsarsem_panel), as well as a demonstration file (demoLMsarsem_panel), to calculate these LM tests have been made available at www.regroningen.nl/elhorst. Alternatively, one may download Matlab files as well as a demonstration file from Donald Lacombe's Web Site www.rri.wvu.edu/lacombe/~lacombe.htm or the Matlab files Debarsy and Ertur (2010) made available at LeSage's Web Site www.spatial-econometrics.com.

If the non-spatial model on the basis of these LM tests is rejected in favor of the spatial lag model or the spatial error model, one should be careful to endorse one of these two models. LeSage and Pace (2009, Ch. 6) recommend to also consider the spatial Durbin model. This model extends the spatial lag model with spatially lagged independent variables

$$y_{it} = \delta \sum_{j=1}^{N} w_{ij} y_{jt} + \alpha + x_{it}\beta + \sum_{j=1}^{N} w_{ij} x_{ijt}\theta + \mu_i \text{(optional)} + \lambda_t \text{(optional)} + \varepsilon_{it}, \qquad (3)$$

where θ, just as β, is a K×1 vector of parameters. This model can then be used to test the hypotheses H_0: $\theta=0$ and H_0: $\theta+\delta\beta=0$. The first hypothesis examines whether the spatial Durbin can be simplified to the spatial lag model, and the second hypothesis whether it can be simplified to the spatial error model (Burridge, 1981). Both tests follow a chi-squared distribution with K degrees of freedom. If the spatial lag and the spatial error model are estimated too, these tests can take the form of a Likelihood Ratio (LR) test. If these models are not estimated, these tests can only take the form of a Wald test. LR tests have the

disadvantage that they require more models to be estimated, while Wald tests are more sensitive to the parameterization of nonlinear constraints (Hayashi, 2000, p.122).

If both hypotheses H_0: $\theta=0$ and H_0: $\theta+\delta\beta=0$ are rejected, then the spatial Durbin best describes the data. Conversely, if the first hypothesis cannot be rejected, then the spatial lag model best describes the data, provided that the (robust) LM tests also pointed to the spatial lag model. Similarly, if the second hypothesis cannot be rejected, then the spatial error model best describes the data, provided that the (robust) LM tests also pointed to the spatial error model. If one of these conditions is not satisfied, i.e. if the (robust) LM tests point to another model than the Wald/LR tests, then the spatial Durbin model should be adopted. This is because this model generalizes both the spatial lag and the spatial error model.

The spatial econometrics literature is divided about whether to apply the specific-to-general approach or the general-to-specific approach (Florax et al., 2003; Mur and Angula, 2009). The testing procedure outlined above mixes both approaches. First, the non-spatial model is estimated to test it against the spatial lag and the spatial error model (specific-to-general approach). In case the non-spatial model is rejected, the spatial Durbin model is estimated to test whether it can be simplified to the spatial lag or the spatial error model (general-to-specific approach). If both tests point to either the spatial lag or the spatial error model, it is safe to conclude that that model best describes the data. By contrast, if the non-spatial model is rejected in favor of the spatial lag or the spatial error model while the spatial Durbin model is not, one better adopts this more general model.

2.1 Bias correction

A detailed explanation as to how a fixed or random effects models extended to include a spatially lagged dependent variable or a spatially autocorrelated error term may be estimated is provided by Elhorst (2010a). The estimation of the fixed effects models is based on the demeaning procedure spelled out in Baltagi (2005). Lee and Yu (2010a) label this procedure the direct approach but show that it will yield biased estimates of (some of) the parameters. Starting with a combined spatial lag/spatial error model, also known as the SAC model (LeSage and Pace, 2009, p.32), and using rigorous asymptotic theory, they analytically derive the size of these biases. If the model contains spatial fixed effects but no time-period fixed effects, the parameter estimate of σ^2 will be biased if N is large and T is fixed. If the model contains both spatial and time-period fixed effects, the parameter

estimates of all parameters will be biased if both N and T are large. By contrast, if T is fixed the time effects can be regarded as a finite number of additional regression coefficients similar to the role of β. On the basis of these findings, Lee and Yu (2010a) propose two methods to obtain consistent results. Instead of demeaning, they propose an alternative procedure to wipe out the spatial (and time-period) fixed effects, which reduces the number of observations available for estimation by one observation for every spatial unit in the sample, i.e., from NT to N(T-1) (or [N-1][T-1]) observations. This procedure is labeled the transformation approach. The second approach Lee and Yu propose to obtain consistent results is a bias correction procedure of the parameters estimates obtained by the direct approach based on maximizing the likelihood function that is obtained under the transformation approach. This paper adopts the bias correction procedure and translates the biases Lee and Yu (2010a) derived for the SAC model to successively the spatial lag model, the spatial error model, and the spatial Durbin model.

First, if the spatial lag, spatial error and spatial Durbin model contain spatial fixed effects but no time-period fixed effects, the parameter estimate $\hat{\sigma}^2$ of σ^2 obtained by the direct approach will be biased. This bias can easily be corrected (BC) by (Lee and Yu, 2010a, Equation 18)

$$\hat{\sigma}^2_{BC} = \frac{T}{T-1}\hat{\sigma}^2 . \tag{4}$$

This bias correction will have hardly have any effect if T is large. However, most spatial panels do not meet this requirement. Mathematically, the asymptotic variance matrices of the parameters of the spatial lag, spatial error, and spatial Durbin model do not change as a results of this bias correction. This is the thrust of the bias correction procedure Lee and Yu (2010a) present as a result of theorem 2 in their paper. Therefore, we may apply the algebraic expressions of the variance matrix when using the direct approach.[3] However, since $\hat{\sigma}^2_{BC}$ replaces $\hat{\sigma}^2$ numerically, the standard errors and thus the t-values of the parameter estimates do change.

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[3] These matrices can be derived from Equation (39) in Lee and Yu (2010a). The variance matrices of the spatial lag model and the spatial error model are also provided by Elhorst (2010a) in equations (C.2.29) and (C.2.33), while the expression for the variance matrix of the spatial Durbin model can be obtained by replacing matrix X in (C.2.29) by [X WX] .

Conversely, if the spatial lag, spatial error and spatial Durbin model contain time-period fixed effects but no spatial fixed effects, the parameter estimate $\hat{\sigma}^2$ of σ^2 obtained by the direct approach can be corrected by

$$\hat{\sigma}^2_{BC} = \frac{N}{N-1}\hat{\sigma}^2.$$

(5)

This bias correction is taken from Lee et al. (2010), who consider a block diagonal spatial weights matrix where each block represents a group of (spatial) units that interact with each other but not with observations in other groups. Since this setup is equivalent to a spatial panel data model with time dummies where spatial units interact with each other within the same time period but not with observations in other time periods, it might also be used here. From Equation (5), it can be seen that this bias correction will hardly have any effect if N is large, as in most spatial panels.

If the spatial lag, spatial error and spatial Durbin model contain both spatial and time-period fixed effects, other parameters need to be bias corrected too. Furthermore, the bias correction in the spatial lag model, the spatial error model, and the spatial Durbin model will be different from each other. The bias correction in the spatial lag model takes the form

$$\begin{bmatrix} \hat{\beta} \\ \hat{\delta} \\ \hat{\sigma}^2 \end{bmatrix}_{BC} = \begin{bmatrix} 1_K \\ 1 \\ \frac{T}{T-1} \end{bmatrix} \circ \left(\begin{bmatrix} \hat{\beta} \\ \hat{\delta} \\ \hat{\sigma}^2 \end{bmatrix} - \frac{1}{N}[-\Sigma(\hat{\beta},\hat{\delta},\hat{\sigma}^2)]^{-1} \begin{bmatrix} 0_K \\ 1 \\ \frac{1}{1-\hat{\delta}} \\ \frac{1}{2\hat{\sigma}^2} \end{bmatrix} \right),$$

(6)

where $\Sigma(\hat{\beta},\hat{\delta},\hat{\sigma}^2)$ represents the expected value of the second-order derivatives of the log-likelihood function multiplied by -1/(NT) (Lee and Yu, 2010a, Equation 53) and the symbol ∘ denotes the element-by-element product of two vectors or matrices (also known as the Hadamard product). Similarly, the bias correction in the spatial error model takes the form

$$\begin{bmatrix} \hat{\beta} \\ \hat{\rho} \\ \hat{\sigma}^2 \end{bmatrix}_{BC} = \begin{bmatrix} 1_K \\ 1 \\ \dfrac{T}{T-1} \end{bmatrix} \circ \left(\begin{bmatrix} \hat{\beta} \\ \hat{\rho} \\ \hat{\sigma}^2 \end{bmatrix} - \dfrac{1}{N}[-\Sigma(\hat{\beta},\hat{\rho},\hat{\sigma}^2)]^{-1} \begin{bmatrix} 0_K \\ 1 \\ 1-\hat{\rho} \\ \dfrac{1}{2\hat{\sigma}^2} \end{bmatrix} \right), \tag{7}$$

and in the spatial Durbin model it takes the form

$$\begin{bmatrix} \hat{\beta} \\ \hat{\theta} \\ \hat{\delta} \\ \hat{\sigma}^2 \end{bmatrix}_{BC} = \begin{bmatrix} 1_K \\ 1_K \\ 1 \\ \dfrac{T}{T-1} \end{bmatrix} \circ \left(\begin{bmatrix} \hat{\beta} \\ \hat{\theta} \\ \hat{\delta} \\ \hat{\sigma}^2 \end{bmatrix} - \dfrac{1}{N}[-\Sigma(\hat{\beta},\hat{\theta},\hat{\delta},\hat{\sigma}^2)]^{-1} \begin{bmatrix} 0_K \\ 0_K \\ 1 \\ 1-\hat{\delta} \\ \dfrac{1}{2\hat{\sigma}^2} \end{bmatrix} \right). \tag{8}$$

The expressions in (6), (7) and (8) are based on Lee and Yu (2010a, Equations 34). Mathematically, the asymptotic variance matrices of the parameters of the spatial lag, spatial error, and spatial Durbin model do not change as a results of the bias correction. This is the thrust of the bias correction procedure Lee and Yu (2010a) present as a result of theorems 4 and 5 in their paper. However, since the bias corrected parameter estimates replace the parameter estimates of the direct approach numerically, the standard errors and t-values of the parameter estimates do change.

2.2 Direct, indirect and spillover effects

Many empirical studies use point estimates of one or more spatial regression models to test the hypothesis as to whether or not spatial spillovers exist. However, LeSage and Pace (2009, p.74) point out that this may lead to erroneous conclusions, and that a partial derivative interpretation of the impact from changes to the variables of different model specifications represents a more valid basis for testing this hypothesis. They demonstrate this using a spatial econometric model in a cross-sectional setting (ibid, pp. 34-40). Below we derive the marginal effects of the explanatory variables in a spatial panel data setting.

If the most general model, the spatial Durbin model, is taken as point of departure and rewritten in vector form as

$$Y_t = (I - \delta W)^{-1} \alpha \iota_N + (I - \delta W)^{-1}(X_t \beta + W X_t \theta) + (I - \delta W)^{-1} \varepsilon_t^*, \tag{9}$$

where the error term ε_t^* covers ε_t and, occasionally, spatial and/or time-period specific effects, the matrix of partial derivatives of the dependent variable in the different units with respect to the kth explanatory variable in the different units (say, x_{ik} for $i=1,\ldots,N$) at a particular point in time is

$$\left[\frac{\partial Y}{\partial x_{1k}} \quad \cdot \quad \frac{\partial Y}{\partial x_{Nk}} \right]_t = \begin{bmatrix} \frac{\partial y_1}{\partial x_{1k}} & \cdot & \frac{\partial y_1}{\partial x_{Nk}} \\ \cdot & \cdot & \cdot \\ \frac{\partial y_N}{\partial x_{1k}} & \cdot & \frac{\partial y_N}{\partial x_{Nk}} \end{bmatrix}_t = (I - \delta W)^{-1} \begin{bmatrix} \beta_k & w_{12}\theta_k & \cdot & w_{1N}\theta_k \\ w_{21}\theta_k & \beta_k & \cdot & w_{2N}\theta_k \\ \cdot & \cdot & \cdot & \cdot \\ w_{N1}\theta_k & w_{N2}\theta_k & \cdot & \beta_k \end{bmatrix}. \tag{10}$$

LeSage and Pace define the direct effect as the average of the diagonal elements of the matrix on the right-hand side of (10), and the indirect effect as the average of either the row sums or the column sums of the non-diagonal elements of that matrix (since the numerical magnitudes of these two calculations of the indirect effect are the same, it does not matter which one is used).[4] Since the matrix on the right-hand side of (10) is independent of the time index t, it can be concluded that these calculations are equivalent to those presented in LeSage and Pace (2009) for a cross-sectional setting.

In the spatial error model ($\theta_k = -\delta\beta_k$), the matrix on the right-hand side of (10) reduces to a diagonal matrix such that each diagonal element equals β_k. This implies that the direct effect of the kth explanatory variable in a spatial error model will be β_k and that the indirect effect will be 0, both just as in a non-spatial model. In the spatial lag model, we have $\theta_k = 0$. Although all non-diagonal elements of the second matrix on the right-hand side of (10) become zero as a results, the direct and indirect effects in the spatial lag model do not reduce to one single coefficient or to zero as in the spatial error model. Consequently, the matrix operations described above to calculate the direct and indirect effects estimates remain necessary.

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[4] The average row effect quantifies the impact on a particular element of the dependent variable as a result of a unit change in all elements of an exogenous variable, while the average column effect quantifies the impact of changing a particular element of an exogenous variable on the dependent variable of all other units.

Although the calculation of the direct and indirect effects is straightforward, one problem is that it cannot be seen from the coefficient estimates and the corresponding standard errors or t-values (derived from the variance-covariance matrix) whether these direct and indirect effects are significant. This is because they are composed of different coefficient estimates according to complex mathematical formulas and the dispersion of these indirect/direct effects depends on the dispersion of all coefficient estimates involved. In order to draw inferences regarding the statistical significance of the direct and indirect effects, LeSage and Pace (2009, p.39) therefore suggest simulating the distribution of the direct and indirect effects using the variance-covariance matrix implied by the maximum likelihood estimates.

One particular parameter combination of δ, β, θ and σ^2 drawn from this variance-covariance matrix (indexed by d) can be obtained by

$$\begin{bmatrix} \delta_d & \beta_d^T & \theta_d^T & \sigma_d^2 \end{bmatrix}^T = P^T \vartheta + \begin{bmatrix} \hat{\delta} & \hat{\beta}^T & \hat{\theta}^T & \hat{\sigma}^2 \end{bmatrix}^T, \tag{11}$$

where P denotes the upper-triangular Cholesky decomposition of $Var(\hat{\delta}, \hat{\beta}, \hat{\theta}, \hat{\sigma}^2)$ and ϑ is a vector of length 2+2K (the number of parameters that have been estimated, leaving the intercept and the fixed effects aside) containing random values drawn from a normal distribution with mean zero and standard deviation one. If D parameter combinations are drawn like this and the (in)direct effect of a particular explanatory variable is determined for every parameter combination, the overall (in)direct effect can be approximated by computing the mean value over these D draws and its significance level (t-value) by dividing this mean by the corresponding standard deviation.

There are two possible approaches to program this. One is to determine the matrix on the right-hand side of (10) for every draw before calculating the direct and indirect effects of these draws. The disadvantage of using this approach is that $(I-\delta W)^{-1}$ needs to be determined for every draw, which will be rather time-consuming and even might break down due to memory problems in case N is large. The other approach, proposed by LeSage and Pace (2009, pp. 114-115), is to use the following decomposition

$$(I - \delta W)^{-1} = I + \delta W + \delta^2 W^2 + \delta^3 W^3 + ..., \tag{12}$$

and to store the traces of the matrices I up to and including W^{100} on the right-hand side of (12) in advance. The calculation of the direct and indirect effects then no longer requires the inversion of the matrix $(I-\delta W)$ for every parameter combination drawn from the variance-covariance matrix in (11), but only a matrix operation based on the stored traces which, as a result, does not require much computational effort. The first approach has been programmed in a separate routine called "direct_indirect_effects_estimates", and the second approach in a separate routine called "panel_effects_estimates". This second routine has been developed and made available by Donald Lacombe (www.rri.wvu.edu/lacombe/~lacombe.htm). If a researcher for whatever reason is not interested in the direct/indirect effects estimates (e.g. in a Monte Carlo simulation experiment), he can save computation time by leaving these calculations aside.

3. Empirical Application

Baltagi and Li (2004) estimate a demand model for cigarettes based on a panel from 46 U.S. states

$$\log(C_{it}) = \alpha + \beta_1 \log(P_{it}) + \beta_2 \log(Y_{it}) + \mu_i (\text{optional}) + \lambda_t (\text{optional}) + \varepsilon_{it}, \tag{13}$$

where C_{it} is real per capita sales of cigarettes by persons of smoking age (14 years and older). This is measured in packs of cigarettes per capita. P_{it} is the average retail price of a pack of cigarettes measured in real terms. Y_{it} is real per capita disposable income. Whereas Baltagi and Li (2004) use the first 25 years for estimation to reserve data for out of sample forecasts, we use the full data set covering the period 1963-1992.[5] Details on data sources are given in Baltagi and Levin (1986, 1992) and Baltagi et al. (2000). They also give reasons to assume the state-specific effects (μ_i) and time-specific effects (λ_t) fixed, in which case one includes state dummy variables and time dummies for each year in equation (13). In this paper we will investigate whether these fixed effects are jointly significant and whether random effects can replace them.

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[5] The dataset can be downloaded freely from www.wiley.co.uk/baltagi/. An adapted version of this dataset is available at www.regroningen.nl/elhorst.

Table 1 reports the estimation results when adopting a non-spatial panel data model and test results to determine whether the spatial lag model or the spatial error model is more appropriate. These results have been obtained and can be replicated by running the demonstration file "demoLMsarsem_panel". When using the classic LM tests, both the hypothesis of no spatially lagged dependent variable and the hypothesis of no spatially autocorrelated error term must be rejected at 5% as well as 1% significance, irrespective of the inclusion of spatial and/or time-period fixed effects. When using the robust tests, the hypothesis of no spatially autocorrelated error term must still be rejected at 5% as well as 1% significance. However, the hypothesis of no spatially lagged dependent variable can no longer be rejected at 5% as well as 1% significance, provided that time-period or spatial and time-period fixed effects are included.[6] Apparently, the decision to control for spatial and/or time-period fixed effects represents an important issue.

<< Table 1 around here >>

To investigate the (null) hypothesis that the spatial fixed effects are jointly insignificant, one may perform a likelihood ratio (LR) test.[7] The results (2315.7, with 46 degrees of freedom [df], $p < 0.01$) indicate that this hypothesis must be rejected. Similarly, the hypothesis that the time-period fixed effects are jointly insignificant must be rejected (473.1, 30 df, $p < 0.01$). These test results justify the extension of the model with spatial and time-period fixed effects, which is also known as the two-way fixed effects model (Baltagi, 2005).

Up to this point, the test results point to the spatial error specification of the two-way fixed effects model. In view of our testing procedure spelled out in Section 2, we now consider the spatial Durbin specification of the cigarette demand model. Its results are reported in columns (1) and (2) of Table 2 and can be replicated by running the demonstration file "demopanelscompare". The first column gives the results when this model is estimated using the direct approach, and the second column when the coefficients are bias corrected according to (8). The results in columns (1) and (2) show that the

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[6] Note that the test results satisfy the condition that LM spatial lag + robust LM spatial error = LM spatial error + robust LM spatial lag (Anselin et al., 1996).
[7] These tests are based on the log-likelihood function values of the different models. Table 1 shows that these values are positive, even though the log-likelihood functions only contain terms with a minus sign. However, since $\sigma^2 < 1$, we have $-\log(\sigma^2) > 0$. Furthermore, since this positive term dominates the negative terms in the log-likelihood function, we eventually have LogL>0.

differences between the coefficient estimates of the direct approach and of the bias corrected approach are small for the independent variables (X) and σ^2. By contrast, the coefficients of the spatially lagged dependent variable (WY) and of the independent variables (WX) appear to be quite sensitive to the bias correction procedure. This is the main reason why it has been decided to build in the bias correction procedure in the Matlab routines dealing with the fixed effects spatial lag and the fixed effects spatial error model (the routines "sar_panel_FE" and "sem_panel_FE"), Furthermore, bias correction is the default option in these SAR and SEM panel data estimation routines, but the user can set an input option (info.bc=0) to turn off bias correction, resulting in uncorrected parameter estimates.

<< Table 2 around here >>

To test the hypothesis whether the spatial Durbin model can be simplified to the spatial error model, H_0: $\theta+\delta\beta=0$, one may perform a Wald or LR test. The results reported in the second column using the Wald test (8.98, with 2 degrees of freedom [df], p=0.011) or using the LR test (8.23, 2 df, p=0.016) indicate that this hypothesis must be rejected. Similarly, the hypothesis that the spatial Durbin model can be simplified to the spatial lag model, H_0: $\theta=0$, must be rejected (Wald test: 14.83, 2 df, p=0.006; LR test: 15.75, 2 df, p=0.004). This implies that both the spatial error model and the spatial lag model must be rejected in favor of the spatial Durbin model.

The third column in Table 2 reports the parameter estimates if we treat μ_i as a random variable rather than a set of fixed effects. These results have been obtained and can be replicated by running the demonstration file "demopanelscompare". Hausman's specification test can be used to test the random effects model against the fixed effects model (see Lee and Yu, 2010b for mathematical details).[8] The results (30.61, 5 df, p<0.01) indicate that the random effects model must be rejected. Another way to test the random effects model against the fixed effects model is to estimate the parameter "phi" (ϕ^2 in Baltagi, 2005), which measures the weight attached to the cross-sectional component of the data and which can take values on the interval [0,1]. If this parameter equals 0, the random

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[8] Mutl and Pfaffermayr (2010) derive the Hausman test when the fixed and random effects models are estimated by 2SLS instead of ML.

effects model converges to its fixed effects counterpart; if it goes to 1, it converges to a model without any controls for spatial specific effects. We find phi=0.087, with t-value of 6.81, which just as Hausman's specification test indicates that the fixed and random effects models are significantly different from each other.

The coefficients of the two explanatory variables in the non-spatial model are significantly different from zero and have the expected signs. In the two-way fixed effects version of this model (the last column of Table 1), higher prices restrain people from smoking, while higher income levels have a positive effect on cigarette demand. The price elasticity amounts to -1.035 and the income elasticity to 0.529. However, as the spatial Durbin model specification of this model was found to be more appropriate, we identify these elasticities as biased. To investigate this, it is tempting to compare the coefficient estimates in the non-spatial model with their counterparts in the two-way spatial Durbin model, but this comparison is invalid. Whereas the parameter estimates in the non-spatial model represent the marginal effect of a change in the price or income level on cigarette demand, the coefficients in the spatial Durbin model do not. For this purpose, one should use the direct and indirect effects estimates derived from equation (10). These effects are reported in the bottom rows of Table 2. The reason that the direct effects of the explanatory variables are different from their coefficient estimates is due to the feedback effects that arise as a result of impacts passing through neighboring states and back to the states themselves. These feedback effects are partly due to the coefficient of the spatially lagged dependent variable [W*Log(C)], which turns out to be positive and significant, and partly due to the coefficient of the spatially lagged value of the explanatory variable itself. The latter coefficient turns out to be negative and significant for the income variable [W*Log(Y)], and to be positive but insignificant for the price variable [W*Log(P)]. The direct and indirect effects estimates and their t-values are computed using the two methods explained in the previous section: the first estimate is obtained by computing $(I-\delta W)^{-1}$ for every draw, while the second estimate is obtained using Equation (12). Since these differences are negligible, we focus on the first numbers below.

In the two-way fixed effects spatial Durbin model (column (2) of Table 2) the direct effect of the income variable appears to be 0.594 and of the price variable to be -1.013. This means that the income elasticity of 0.529 in the non-spatial model is underestimated by 10.9% and the price elasticity of -1.035 by 2.1%. Since the direct effect of the income

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variable is 0.594 and its coefficient estimate 0.601, its feedback effect amounts to -0.007 or -1.2% of the direct effect. Similarly, the feedback effect of the price variable amounts to -0.012 or 1.2% of the direct effect. In other words, these feedback effects turn out to be relatively small. By contrast, whereas the indirect effects in the non-spatial model are set to zero by construction, the indirect effect of a change in the explanatory variables in the spatial Durbin model appears to be 21.7% of the direct effect in case of the price variable and -33.2% in case of the income variable. Furthermore, based on the t-statistics calculated from a set of 1,000 simulated parameter values, these two indirect effects appear to be significantly different from zero. In other words, if the price or the income level in a particular state increases, not only cigarette consumption in that state itself but also in that of its neighboring states will change; the change in neighboring states to the change in the state itself is in the proportion of approximately 1 to 4.6 in case of a price change and 1 to -3.0 in case of an income change.

Up to now, many empirical studies used point estimates of one or more spatial regression model specifications to test the hypothesis as to whether or not spatial spillover exist. The results above illustrate that this may lead to erroneous conclusions. More specifically, whereas the coefficient of the spatial lagged value of the price variable is positive and insignificant, its indirect or spillover effect is negative and significant.

The finding that own-state price increases will restrain people not only from buying cigarettes in their own state (elasticity -1.01) but to a limited extent also from buying cigarettes in neighboring states (elasticity -0.22) is not consistent with Baltagi and Levin (1992). They found that price increases in a particular state —due to tax increases meant to reduce cigarette smoking and to limit the exposure of non-smokers to cigarette smoke— encourage consumers in that state to search for cheaper cigarettes in neighboring states. Since Baltagi and Levin (1992) estimate a dynamic but non-spatial panel data model, an interesting topic for further research is whether our spatial spillover effect would change sign when considering a dynamic spatial panel data model. LeSage and Pace (2009, Ch. 7) and Parent and LeSage (2010) find that dynamic spatial panel data models with relatively high temporal dependence and low spatial dependence may correspond to cross-sectional spatial regressions or to static spatial panel data regressions with relatively high spatial dependence. Whether such an empirical relationship also exists for cigarette demand is another interesting topic for further research.

The results reported in Table 2 illustrate that the t-values of the indirect effects compared to those of the direct effects are relatively small, -24.73 versus -2.26 for the price variable and 10.45 versus -2.15 for the income variable. Experience shows that one needs quite of lot of observations over time to find significant coefficient estimates of the spatially lagged independent variables and, related to that, significant estimates of the indirect effects. It is one of the obstacles to the spatial Durbin model in empirical research. Since most practitioners use cross-sectional data or panel data over a relatively short period of time, they often cannot reject the hypothesis that the coefficients of the spatially lagged independent variables are jointly insignificant (H_0: $\theta=0$), as a result of which they are inclined to accept the spatial lag model. However, one important limitation of the spatial lag model is that the ratio between the direct and indirect effects is the same for every explanatory variable by construction (Elhorst, 2010b). In other words, whereas we find that the ratio between the indirect and the direct effects is positive and significant for the price variable (21.7%) and negative and significant (-33.2%) for the income variable, these percentages can not be different from each other when adopting the spatial lag model. In this case, both would amount to approximately 27.1%. Therefore, practitioners should think twice before abandoning the spatial Durbin model, since not only significance levels count but also flexibility.

4. Conclusions

This paper presents Matlab software to estimate spatial panel data models, among which the spatial lag model, the spatial error model, and the spatial Durbin model extended to include spatial and/or time-period fixed effects or extended to include spatial random effects. These routines now also feature:

1. A generalization of the classic and the robust LM tests to a spatial panel data setting based on Debarsy and Ertur (2010);

2. The bias correction procedure proposed by Lee and Yu (2010a) if the spatial panel data model contain spatial and/or time-period fixed effects;

3. The direct and indirect effects estimates of the explanatory variables proposed by LeSage and Pace (2009);

4. A framework to test the spatial Durbin model against the spatial lag and the spatial error model;

5. A framework to choose among fixed effects, random effects or a model without fixed/random effects.

According to Anselin (2010), spatial econometrics has reached a stage of maturity through general acceptance of spatial econometrics as a mainstream methodology; the number of applied empirical researchers who use econometric techniques in their work also indicates nearly exponential growth. The availability of more and better software, not only for cross-sectional data but also for spatial panels and not only written in Matlab but also in easier accessible packages such as Stata, might encourage even more researchers to enter this field.

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Table 1. Estimation results of cigarette demand using panel data models without spatial interaction effects

Determinants	(1)	(2)	(3)	(4)
	Pooled OLS	Spatial fixed effects	Time-period fixed effects	Spatial and time-period fixed effects
Log(P)	-0.859	-0.702	-1.205	-1.035
	(-25.16)	(-38.88)	(-22.66)	(-25.63)
Log(Y)	0.268	-0.011	0.565	0.529
	(10.85)	(-0.66)	(18.66)	(11.67)
Intercept	3.485			
	(30.75)			
σ^2	0.034	0.007	0.028	0.005
R^2	0.321	0.853	0.440	0.896
LogL	370.3	1425.2	503.9	1661.7
LM spatial lag	66.47	136.43	44.04	46.90
LM spatial error	153.04	255.72	62.86	54.65
robust LM spatial lag	58.26	29.51	0.33	1.16
robust LM spatial error	144.84	148.80	19.15	8.91

Notes: t-values in parentheses.

Table 2. Estimation results of cigarette demand: spatial Durbin model specification with spatial and time-period specific effects

Determinants	(1)		(2)		(3)	
	Spatial and time-period fixed effects		Spatial and time-period fixed effects bias-corrected		Random spatial effects, Fixed time-period effects	
W*Log(C)	0.219	(6.67)	0.264	(8.25)	0.224	(6.82)
Log(P)	-1.003	(-25.02)	-1.001	(-24.36)	-1.007	(-24.91)
Log(Y)	0.601	(10.51)	0.603	(10.27)	0.593	(10.71)
W*Log(P)	0.045	(0.55)	0.093	(1.13)	0.066	(0.81)
W*Log(Y)	-0.292	(-3.73)	-0.314	(-3.93)	-0.271	(-3.55)
phi					0.087	(6.81)
σ^2		0.005		0.005		0.005
R^2		0.901		0.902		0.880
Corrected R^2		0.400		0.400		0.317
LogL		1691.4		1691.4		NA
Wald test spatial lag		14.83 (p=0.006)		17.96 (p=0.001)		13.90 (p=0.001)
LR test spatial lag		15.75 (p=0.004)		15.80 (p=0.004)		NA
Wald test spatial error		8.98 (p=0.011)		8.18 (p=0.017)		7.38 (p=0.025)
LR test spatial error		8.23 (p=0.016)		8.28 (p=0.016)		NA
Direct effect Log(P)	-1.015	-1.014	-1.013	-1.012	-1.018	-1.018
	(-24.34)	(-25.44)	(-24.73)	(-23.93)	(-24.64)	(-25.03)
Indirect effect Log(P)	-0.210	-0.211	-0.220	-0.215	-0.199	-0.195
	(-2.40)	(-2.37)	(-2.26)	(-2.12)	(-2.28)	(-2.19)
Total effect Log(P)	-1.225	-1.225	-1.232	-1.228	-1.217	-1.213
	(-12.56)	(-12.37)	(-11.31)	(-11.26)	(-12.43)	(-12.21)
Direct effect Log(Y)	0.591	0.594	0.594	0.594	0.586	0.583
	(10.62)	(10.44)	(10.45)	(10.67)	(10.68)	(10.53)
Indirect effect Log(Y)	-0.194	-0.194	-0.197	-0.196	-0.169	-0.171
	(-2.29)	(-2.27)	(-2.15)	(-2.18)	(-2.03)	(-2.06)
Total effect Log(Y)	0.397	0.400	0.397	0.398	0.417	0.412
	(5.05)	(5.19)	(4.61)	(4.62)	(5.45)	(5.37)

Notes: t-values in parentheses. Direct and indirect effects estimates: Left column $(I-\delta W)^{-1}$ computed every draw, right column $(I-\delta W)^{-1}$ calculated by Equation (12). Corrected R^2 is R^2 without the contribution of fixed effects.